OCSim Modules (Optical Receivers: Shot Noise, Thermal Noise and Signal-to-Noise Ratio of Direct Detection Optical Receivers)

SCIENTIFIC MANUAL

MODULE 13: OPTICAL RECEIVERS: SHOT NOISE, THERMAL NOISE AND SIGNAL-TO-NOISE RATIO OF DIRECT DETECTION OPTICAL RECEIVERS

CodeSScientific

OCSim Advanced Level Software Modules

Softwares for Fiber Optic Communication Systems

Module 13: Optical Receivers: Shot Noise, Thermal Noise and Signal-to-Noise Ratio of Direct Detection Optical Receivers

Scientific Manual

Background Theory and Formulation of the Module

CodeSScientific

sales@codesscientific.com / www.codesscientific.com

Trademarks

Disclaimer

Optical Receivers

Background Theory and Formulation of the Module

Receiver Noise

An optical receiver translates the data in the optical domain into electrical domain, but it also adds noise. Two important noise sources are:

(i) Shot noise

(ii) Thermal noise

Shot noise

For a non-ideal photodiode (with quantum efficiency $\eta < 1$), the probability of an event that a photon incident on the photodiode generates an electron-hole pair that contributes to the photocurrent is η or the probability of this even not happening is $1 - \eta$. The randomly generated photo carriers lead to fluctuations in photocurrent which is known as *shot noise*. When an optical signal of power P_I falls on the *pin* photo-diode, the current can be written as,

$$I(t) = I_{PC} + i_{shot}(t) \quad (1)$$

Where $I_{PC} = RP_I$ *is* the deterministic part of the current, $i_{shot}\ (t)$ is the noise component of the current due to shot noise. The shot noise current $i_{shot}\ (t)$ is a random variable with zero mean, i.e., $< i_{shot}\ (t) >= 0$. The noise power dissipated due to $i_{shot}\ (t)$ in a 1 Ω resistor is $i^2_{shot}(t)$. The mean noise power is,

$$N^1_{shot} =< i^2_{shot}(t) >= \sigma^2_{shot} \tag{2}$$

Where σ_{shot} the standard deviation and superscript 1 is indicates that it is the noise power dissipated in a 1 Ω resistor. Thus, the variance is the same as the mean noise power dissipated in a 1 Ω resistor.

The shot noise is a white noise process and its power spectral density is constant. The power spectral density (PSD) of the shot noise is,

$$\sigma_{shot}(f) = q\, I_{PC} \tag{3}$$

Where q is the electron charge.

For simplicity, let us assume that the receiver is an ideal low pass filter with bandwidth, B_e. The receiver transfer equation $\widetilde{H}_e\ (f)$ is given by,

$$\widetilde{H}_e\ (f) = \begin{cases} 1, & if\ |f| < B_e \\ 0, & otherwise \end{cases} \tag{4}$$

The shot noise generated in the photodiode passes through the rest of the receiver circuit. The PSD at the receiver output is,

$$\sigma_{shot,out}\,(f) = \sigma_{shot}\,(f)\,\left|\widetilde{H}_e\,(f)\right|^2 \tag{5}$$

The PSD refers to the mean power per unit frequency interval and therefore, the mean noise power at the receiver output (dissipated in a 1 Ω resistor) can be obtained by integrating the PSD over frequency,

$$N_{shot}^1 = \int_{-\infty}^{\infty} \sigma_{shot,out}\; f df \tag{6}$$

Using Eqs. (3) and (4) in (6), we obtain,

$$\begin{aligned} N_{shot}^1 &= qI_p \int_{-\infty}^{\infty} \left|\widetilde{H}_e\,(f)\right|^2 df \\ &= 2qI_{PC} \end{aligned} \tag{7}$$

Using Eq. (2), we find the variance of shot noise as,

$$\sigma_{shot}^2 = N_{shot}^1 = 2qI_{PC}\,B_e \tag{8}$$

Eq. (8) is valid for arbitrary filter shapes if the effective bandwidth B_e is defined as,

$$B_e = \frac{1}{2}\int_{-\infty}^{\infty}\left|\widetilde{H}_e\,(f)\right|^2 df \tag{9}$$

When the dark current I_d is not negligible, Eq. (8) is modified as,

$$\sigma_{shot}^2 = 2q\,(I_{PC} + I_d\,)B_e \tag{10}$$

For APD receivers, the variance of shot noise is given by,

$$\sigma_{shot}^2 = 2qM^2F\,(RP_1 + I_d\,)B_e \tag{11}$$

Where *M* is the multiplication factor and *F* is the excess noise factor.

Thermal Noise

Electrons move randomly in a conductor. As the temperature increases, electrons move faster and therefore electron current increases. However, the mean value of the current is zero since, on the average, there are as many electrons moving in one direction as there are in another. Because of the random motion of electrons, the resulting current is noisy and as called 'thermal noise' or 'Johnson noise'.

In the presence of thermal noise, current in the receiver circuit may be written as,

$$I(t) = I_{PC} + i_{thermal}\,(t) \tag{12}$$

Where I_{PC} is the mean photo current (deterministic), and $i_{thermal}(t)$ is the thermal noise current.

For low frequencies $(f \ll kBT = h)$, thermal noise can be regarded as white noise, i.e. its power spectral density is constant. It is given by,

$$\rho_{thermal}(f) = 2k_B T / R_L \tag{13}$$

Where k_B is the Boltzmann's constant, R_L is the load resistance and T is the absolute temperature. If B_e is the effective bandwidth of the receiver, the noise variance can be calculated as before,

$$\sigma_{thermal}^2 = < i_{thermal}^2 > = 4k_B T B_e / R_L \tag{14}$$

Eq. (14) does not include the noise sources in the amplifier circuit such as that coming from resistors and active elements. Eq. (14) can be modified to account for the noise sources within the amplifier as,

$$\sigma_{thermal}^2 = 4k_B T B_e F_n / R_L \tag{15}$$

Where F_n is the amplifier noise factor.

Signal to Noise Ratio (SNR)

Let us first consider *pin* receivers. The mean signal power is,

$$S = I_{PC}^2 R_L = (RP_I)^2 R_L \tag{16}$$

Using Eq. (10), the mean noise power dissipated in the resistor R_L due to the shot noise current is,

$$N_{shot} = < i_{shot}^2(t) > R_L = 2q(I_{PC} + I_d)B_e R_L \tag{17}$$

Using Eq. (15), the mean noise power due to thermal noise is,

$$N_{thermal} = < i_{thermal}^2 R_L > = 4k_B T B_e F_n \tag{18}$$

The total mean noise power is,

$$N = N_{shot} + N_{thermal} = (\sigma_{shot}^2 + i_{thermal}^2) R_L \tag{19}$$

The signal to noise ratio (SNR) is defined as,

$$S\,N\,R_{pin} = Mean\ Signal\ Power/Mean\ Noise\ Power$$

$$= S/N = I_{PC}^2/(\sigma_{shot}^2 + \sigma_{thermal}^2)$$

$$= R^2 P_I^2/[2q(RP_I + I_d) + 4k_B T F_n/R_L]B_e \tag{20}$$

For APD receivers, the signal power is,

$$S = (MRP_I)^2 R_L \tag{21}$$

Using Eq. (11), SNR can be calculated as,

$$S\,N\,R_{APD} = (MRP_I)^2/[2qM^2F(RP_I + \ I_d) + 4k_BTF_n/R_L]B_e \qquad (22)$$

Company Researchers & Developers

Integrate the Modules with your in-house and Commercial Software & Hardware Products

(1) Modify the Source Code Modules / Components to the Next Level for Your Research Papers, Research Projects and Theses.

(2) Integrate Different Source Code Modules / Components in the OCSim Package to Realize Your Own Fiber Optic Communication Systems.

(3) Use the Existing Source Code Modules / Components for Your Research Papers, Research Projects and Theses.

(4) Use the Existing Source Code Modules for Laboratory Simulation Experiments and Exercises.

Simulations of Shot Noise, Thermal Noise and Signal-to-Noise Ratio of Direct Detection Optical Receivers

Source Code File

Main File : dd_rx_snr.m

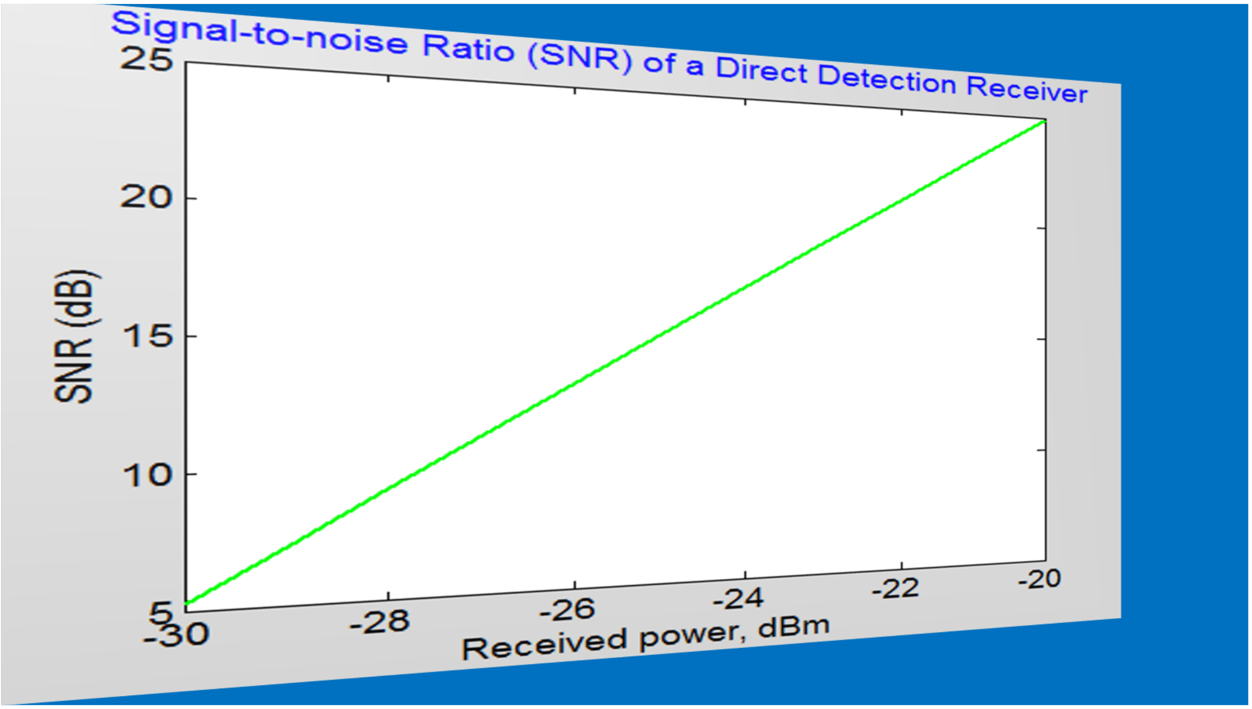
This module simulates the signal-to-noise ratio (SNR) of a direct detection receiver. The variance of shot noise and thermal noise are first calculated and then SNR is calculated for a PIN photodiode.

Explore Further this Module:

1. **Plot** the SNR as a function of received power for different values of the load resistance R_L. **Explain** the dependence of SNR on R_L.
2. **Change** the bit rate from 10 Gb/s to 40 Gb/s. **Plot** the SNR as a function of bit rate for a fixed value of received power. **Why** does the SNR decrease as the bit rate increases?
3. **Modify** this source code program to calculate the SNR of APD receivers.

Selected Simulated Result Using this Module

Simulation of Signal-to-noise Ratio (SNR) of a Direct Detection Receiver



Contact Us for More Details

CodeSScientific Photonics

sales@codesscientific.com / www.codesscientific.com