OCSim Modules
(EM Waves: Simulation of Standing Waves)

MODULE 3: EM WAVES:SIMULATION OF STANDING WAVES

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OCSim Advanced Level Software Modules

Softwares for Fiber Optic Communication Systems

Module 3: EM Waves: Simulation of Standing Waves

Scientific Manual

Background Theory and Formulation of the Module

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Electromagnetics

Background Theory and Formulation of the Module

Maxwell's Equation in Source-Free Region

$$\nabla . \boldsymbol{D} = 0 \tag{1}$$

$$\nabla . \boldsymbol{B} = 0 \tag{2}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \tag{3}$$

$$\nabla \times \boldsymbol{H} = -\frac{\partial \boldsymbol{D}}{\partial t} \tag{4}$$

Electromagnetic Wave

Suppose the electric field is only along x-direction,

$$\mathbf{E} = E_x \mathbf{x} \tag{5}$$

and magnetic field is only along y-direction,

$$\mathbf{H} = H_y \mathbf{y} \tag{6}$$

Substituting Eqs. (5) and (6) into Maxwell′s equations, we find,

$$\frac{\partial^2 E_x}{\partial z^2} = \mu \epsilon \frac{\partial^2 E_x}{\partial t^2} \tag{7}$$

.

The above equation is called the *wave equation* and it forms the basis for the study of electromagnetic wave propagation.

Free Space Propagation

For free space, $\epsilon = \epsilon_0 = 8.854 \times 10^{-12} \mathrm{C}^2/\mathrm{Nm}^2, \mu = \mu_0 = 4\pi \times 10^{-7} \mathrm{N}/\mathrm{A}^2$, and

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \cong 3 \times 10^8 m/s \tag{8}$$

where c is the velocity of light in free space.

Propagation in a Dielectric Medium

Similar to Eq. (8), velocity of light in a medium can be written as,

$$v = \frac{1}{\sqrt{\mu\epsilon}} \tag{9}$$

where $\mu = \mu_0 \mu_r$ and $\epsilon = \epsilon_0 \epsilon_r$. Therefore,

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0 \mu_r \epsilon_r}} \tag{10}$$

Using Eq. (9) in Eq. (10), we have,

$$v = \frac{c}{\sqrt{\mu_r \epsilon_r}} \tag{11}$$

For dielectrics, μ_r= 1 and velocity of light in a dielectric medium can be written as,

$$v = \frac{c}{\sqrt{\epsilon_r}} = \frac{c}{n} \tag{12}$$

where $\sqrt{\epsilon_r}$ is called the refractive index of the medium. The refractive index of a medium is greater than 1 and velocity of light in a medium is less than that in free space.

1-Dimensional Wave Equation

Using Eq. (9) in Eq. (7), we obtain,

$$\frac{\partial^2 E_x}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 E_x}{\partial t^2} \tag{13}$$

To solve Eq. (13), let us try a trial solution of the form,

$$E_x(t, z) = f(t + \alpha z) \tag{14}$$

where f is an arbitrary function of $t + \alpha z$. Let,

$$u = t + \alpha z \tag{15}$$

$$\frac{\partial u}{\partial z} = \alpha, \qquad \frac{\partial u}{\partial t} = 1 \tag{16}$$

$$\frac{\partial E_x}{\partial z} = \frac{\partial E_x}{\partial u}\frac{\partial u}{\partial z} = \frac{\partial E_x}{\partial u}\alpha \tag{17}$$

$$\frac{\partial^2 E_x}{\partial z^2} = \frac{\partial^2 E_x}{\partial u^2}\alpha^2 \tag{18}$$

$$\frac{\partial^2 E_x}{\partial t^2} = \frac{\partial^2 E_x}{\partial u^2} \tag{19}$$

Using Eqs. (18) and (19) in Eq. (13), we obtain,

$$\alpha^2 \frac{\partial^2 E_x}{\partial u^2} = \frac{1}{v^2}\frac{\partial^2 E_x}{\partial u^2} \tag{20}$$

Therefore,

$$\alpha = \pm\frac{1}{v} \tag{21}$$

$$E_x = f\left(t + \frac{z}{v}\right) \text{ or } E_x = f\left(t - \frac{z}{v}\right) \tag{22}$$

The negative sign implies a forward propagating wave and the positive sign indicates a backward propagating wave. Note that f is an arbitrary function and it is determined by the initial conditions as illustrated by the following examples.

Example 1.

Turn on the flash light for 1 ms and turn it off. You will generate a pulse shown in Fig.1 at the flash light ($z = 0$). The electric field intensity oscillates at light frequencies and the rectangular shape shown in Fig. 1 is actually the absolute field

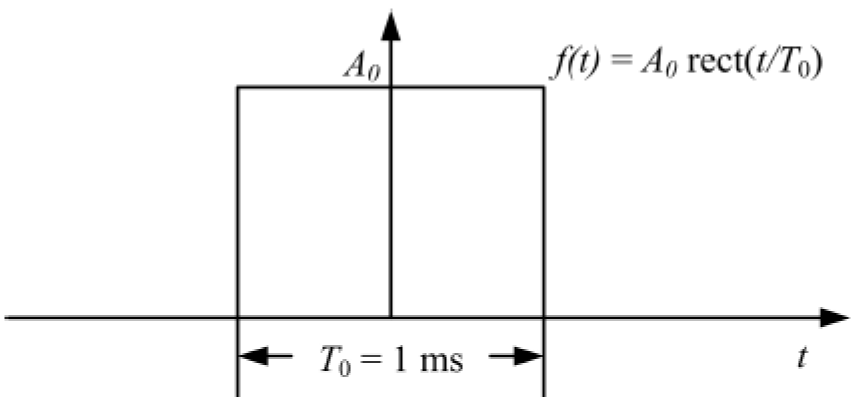


Figure 1. Electrical field $E_x(t, 0)$ at the flash light

envelope. Let us ignore the fast oscillations in this example and write the field (which is actually the field envelope at $z = 0$ as,

$$E_x(t, 0) = f(t) = A_0 \operatorname{rect}\left(\frac{t}{T_0}\right) \tag{23}$$

where

$$\mathrm{rect}(x) = f(x) = \begin{cases} 1, & |x| < \dfrac{1}{2} \\ 0, & \text{otherwise} \end{cases} \quad (24)$$

and $T_0 = 1$ ms. The speed of light in free space, $\mathrm{v} = \mathrm{c} \simeq 3 \times 10^8 m/s$. Therefore, it

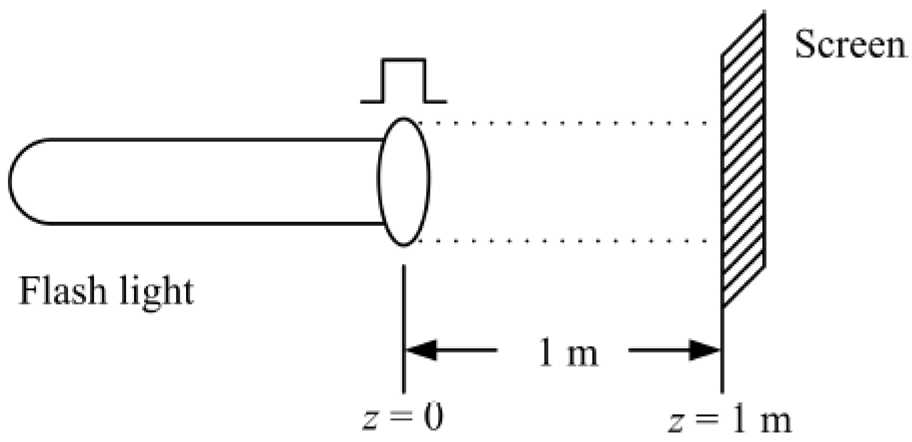


Figure 2. The propagation of the light pulse generated at the flash light.

takes $0.33 \times 10^{-8} s$ to get the light pulse on the screen. At $z = 1$ m,

$$E_x(t, z) = f\left(t - \frac{z}{v}\right) = A_0 \,\mathrm{rect}\left(\frac{t - 0.33 \times 10^{-8}}{T_0}\right) \quad (25)$$

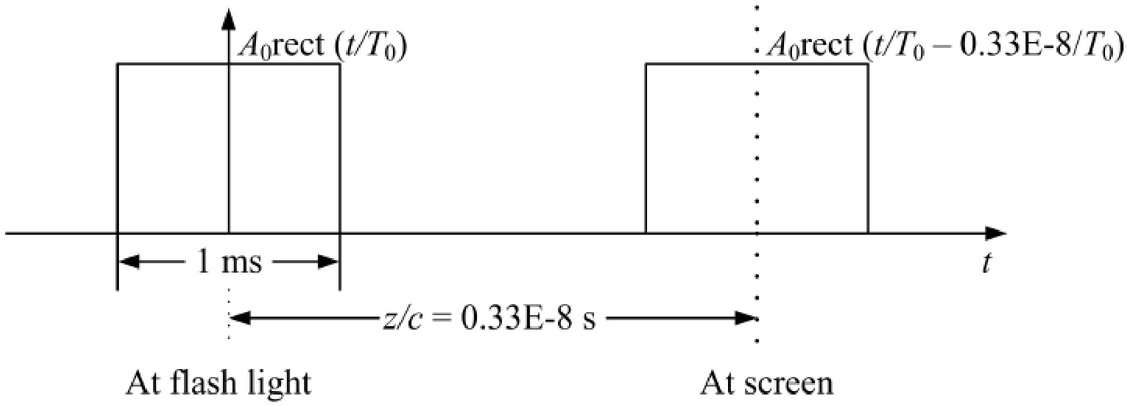


Figure 3. The electric field envelopes at the flash light and the screen.

Example 2.

A laser operates at 191 *THz*. Under ideal conditions and ignoring transverse distributions, the laser output may be written as,

$$E_x(t, 0) = f(t) = A_0 \cos(2\,\pi\, f_0 t) \qquad (26)$$

where f_0= 191 THz. The laser output arrives at the screen after 0.33×10^{-8}. The

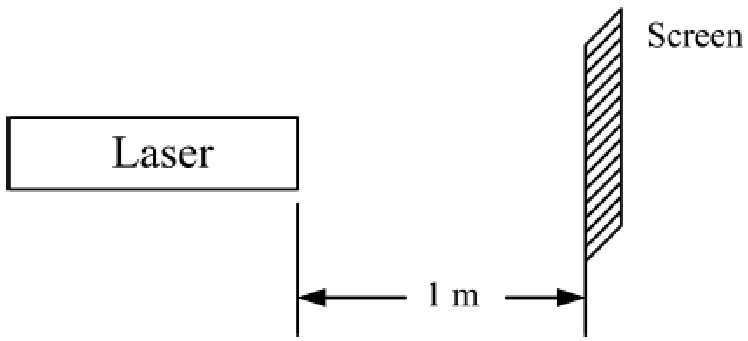


Figure 4. The propagation of laser output in free space.

electric field intensity at the screen may be written as,

$$E_x(t,z) = f\left(t - \frac{z}{v}\right)$$

$$= A\cos\left[2\,\pi\, f_0\left(t - \frac{z}{v}\right)\right]$$

$$= A\cos[2\,\pi\, f_0(t - 0.33 \times 10^{-8})] \qquad (27)$$

Example 3.

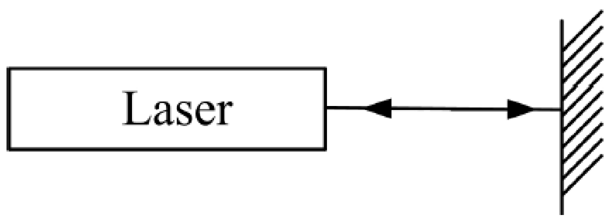


Figure 5. Reflection of the laser output by a mirror.

The laser output is reflected by a mirror and it propagates in backward direction as shown in Fig. 5. In Eq. (22), the positive sign corresponds to backward propagating wave. Suppose that at the mirror electromagnetic wave undergoes a phase shift of $\emptyset$. The backward propagating wave can be described by (see Eq. (22)),

$$E_{x^-} = Acos[2\pi f_0(t + z/v) + \emptyset] \tag{28}$$

The forward propagating wave is described by Eq. (27),

$$E_{x^+} = Acos[2\pi f_0(t - z/v)] \tag{29}$$

Total field is given by,

$$E_x = E_{x^-} + E_{x^+} \tag{30}$$

Company Researchers & Developers

Integrate the Modules with your in-house and Commercial Software & Hardware Products

(1) **Use the Existing Modules** / Components for Your Research & Development.

(2) **Modify** the Modules / Components to the Next Level for Your Research & Development.

(3) **Integrate** Different Modules / Components in the OCSim Package to Realize Your Own Fiber Optic Communication Systems.

(4) **Modify** the Modules for Co-Simulations with the Third-Party Commercial Optical Communication Systems Softwares.

Simulation of Propagation of Cosine Waves

Source Code File

Main File : standing.m

This module simulates standing waves.

Explore Further this Module:

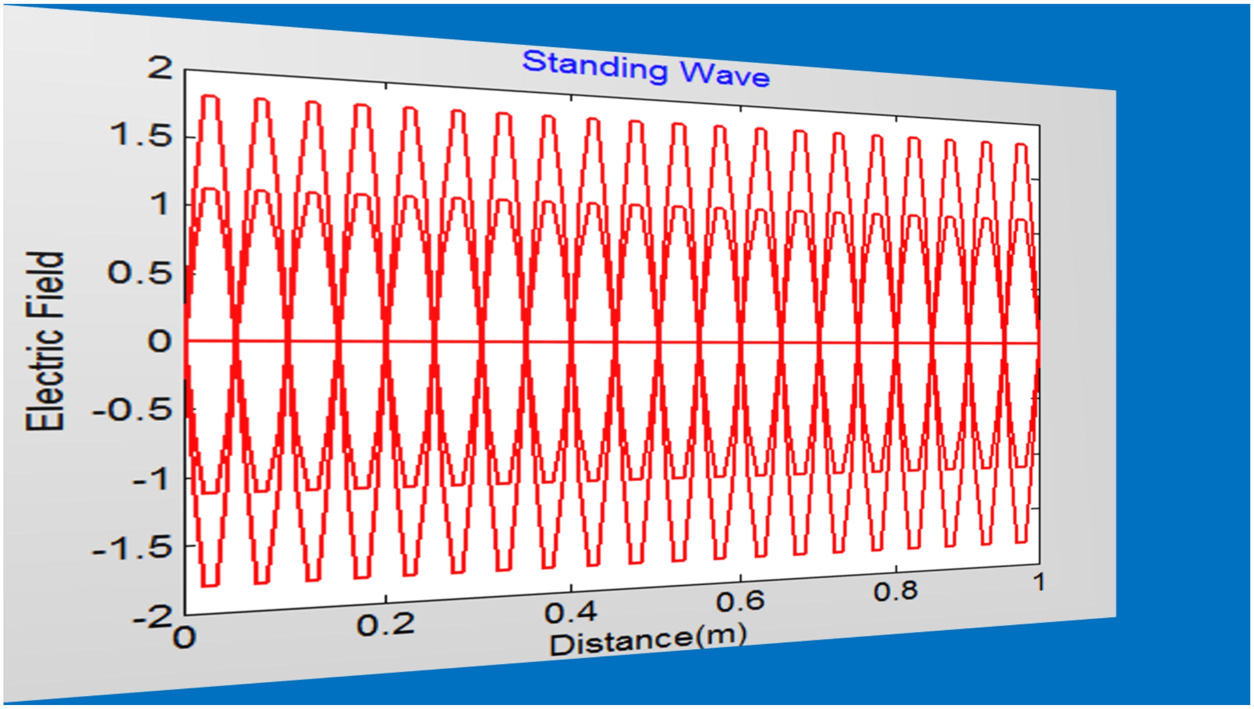
1. A mirror is inserted at the laser end so that light is trapped between mirrors. The mirrors are assumed to be perfect and each introduces a phase shift of π. After a few trips steady state is attained and the field plotted here corresponds to the steady state. The distance between mirrors is assumed to be an integral multiple of half-wavelength. Electric field as a function of propagation distance can be plotted for various time-steps. **Observe** the nodes and anti-nodes. Note that the field at a node is always zero.

2. **Modify** the source code (standing.m) by changing the mode number from 10 to 20 and **plot** the field. **Count** the number of nodes.

3. **Modify** the source code (standing.m) by choosing the distance between mirrors that does not correspond to an integral multiple of half-wavelength. Do you observe nodes and anti-nodes?

Selected Simulated Results Using this Module

Simulation of Standing Waves



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